Statistical Model Checking as Feedback Control

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Rigorous Systems Engineering

Analysis of CPS: Challenges

State-space explosion:

- **Open, physical part, uncertain and distributed**

Model is generally not known:

- **Basic laws of physical part (or controller) only partially available**

Current-state is generally not known:

- **Output is a function of only a subset of the state variables**

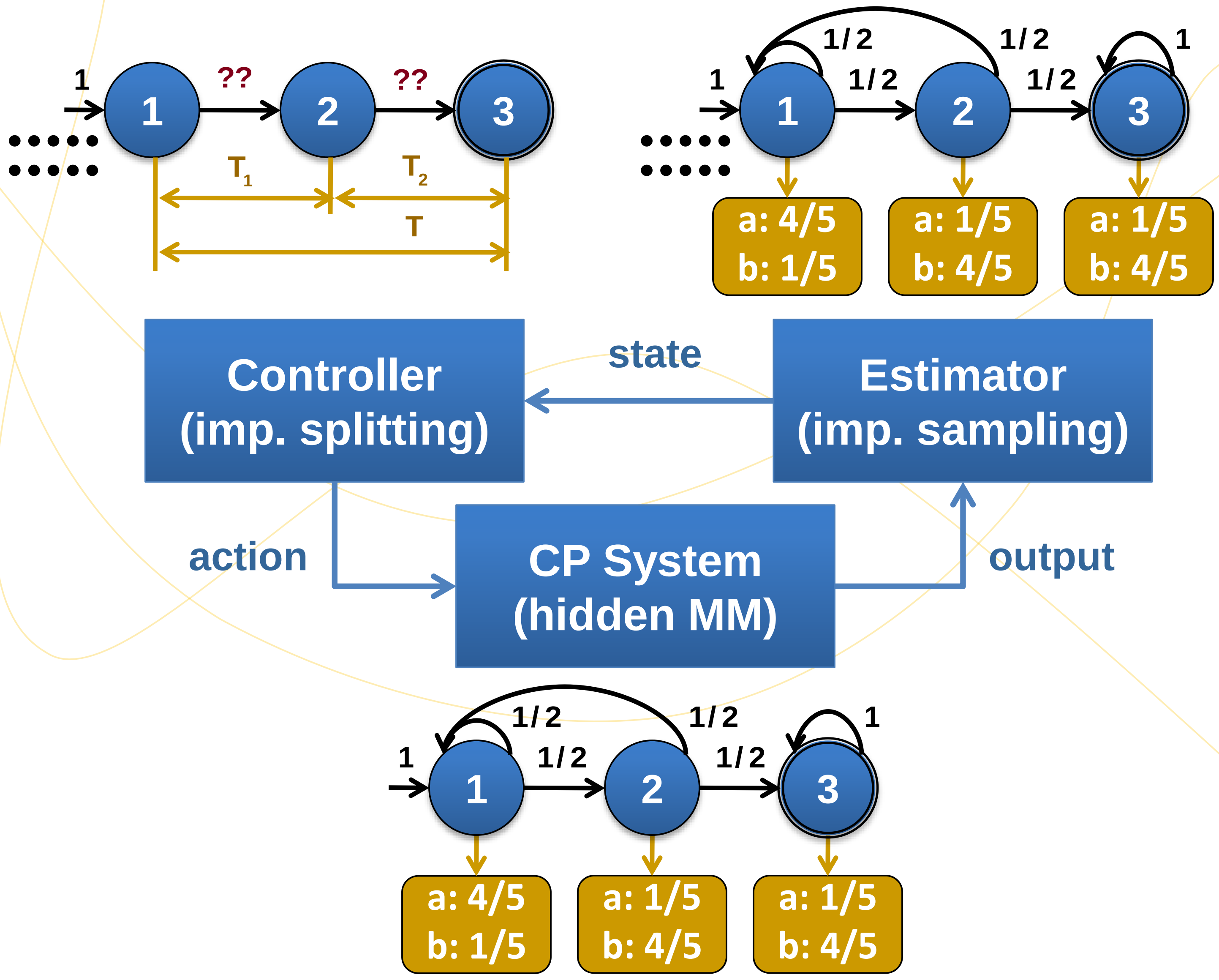
How to steer towards rare events (RE) is a challenge:

- **Relation between RE and the CPS behavior is not known**

Outline

- Learning
- State Estimation
- Control
- Future

Model Checking as Feedback Control

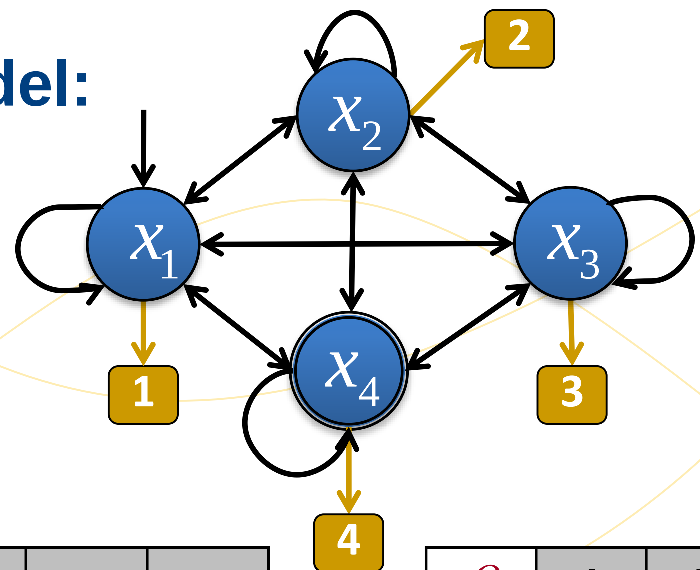


Learning a DTMC: Input

Trace(s): 1, 1, 2, 3, 1, 2, 2, 2, 3, 3, 1, 2, 3, 3, 3, ...

Unknown model:

assume DTMC

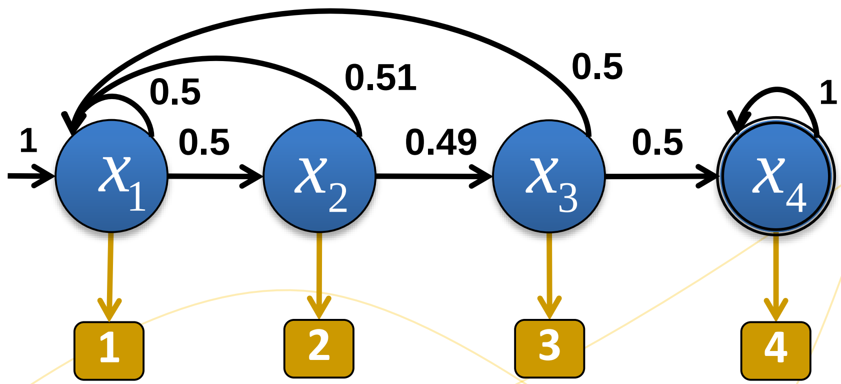


T_H	x_1	x_2	x_3	x_4
x_1	0.25	0.25	0.25	0.25
x_2	0.25	0.25	0.25	0.25
x_3	0.25	0.25	0.25	$\mathbf{P}(x_4 \mid x_3)$
x_4	0.25	0.25	0.25	0.25

O_H	1	2	3	4
x_1	1	0	0	0
x_2	0	1	0	0
x_3	0	0	1	$\mathbf{P}(4 \mid x_3)$
x_4	0	0	0	1

Learning a DTMC: Output

Learned model



T_H	x_1	x_2	x_3	x_4
x_1	0.5	0.5	0	0
x_2	0.51	0	0.49	0
x_3	0.5	0	0	0.5
x_4	0	0	0	1

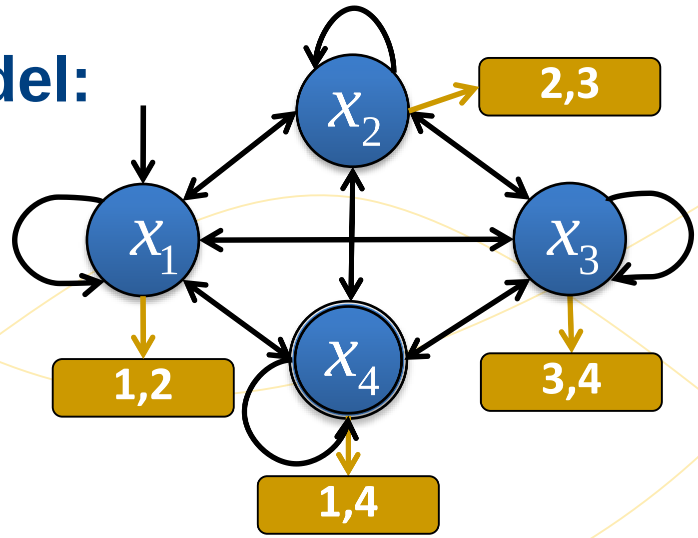
O_H	1	2	3	4
x_1	1	0	0	0
x_2	0	1	0	0
x_3	0	0	1	0
x_4	0	0	0	1

Learning a DTMC: Input

Trace(s): 1, 1, 2, 3, 1, 2, 2, 2, 3, 3, 1, 2, 3, 3, 3, ...

Unknown model:

assume HMM

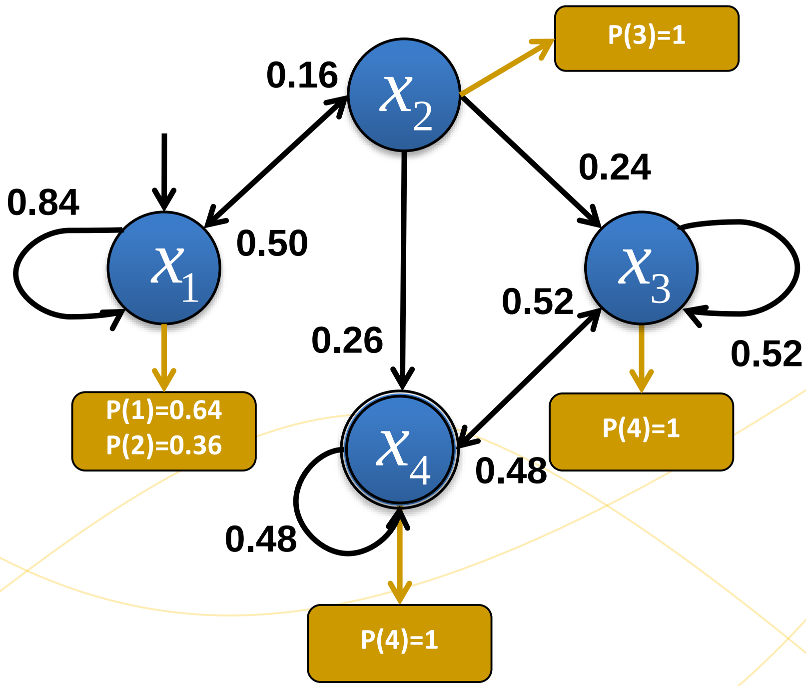


T_H	x_1	x_2	x_3	x_4
x_1	0.25	0.25	0.25	0.25
x_2	0.25	0.25	0.25	0.25
x_3	0.25	0.25	0.25	0.25
x_4	0.25	0.25	0.25	0.25

O_H	1	2	3	4
x_1	0.5	0.5	0	0
x_2	0	0.5	0.5	0
x_3	0	0	0.5	0.5
x_4	0.5	0	0	0.5

Learning a DTMC: Output

Learned model

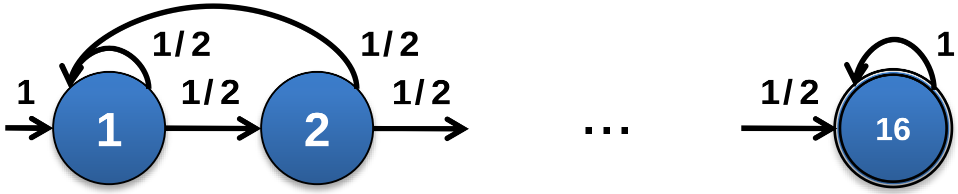


Initial O_H big impact!

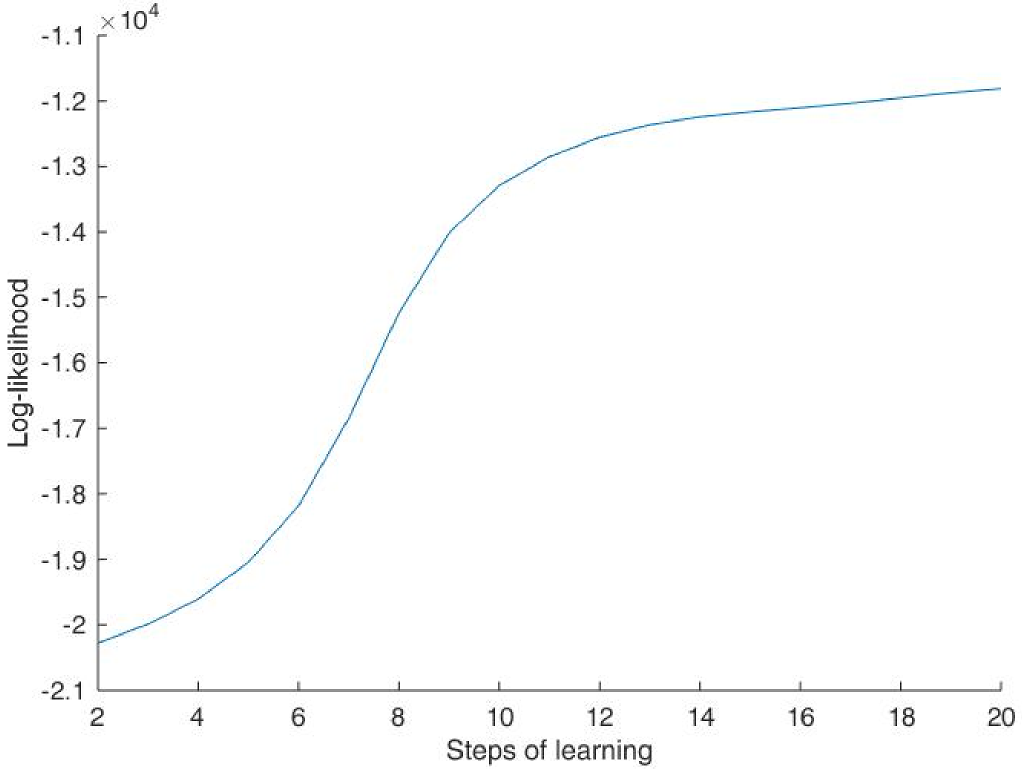
T_H	x_1	x_2	x_3	x_4
x_1	0.84	0.16	0	0
x_2	0.50	0	0.24	0.26
x_3	0	0	0.52	0.48
x_4	0	0	0.52	0.48

O_H	1	2	3	4
x_1	0.67	0.33	0	0
x_2	0	0	1	0
x_3	0	0	0	1
x_4	0	0	0	1

Discrete-time Markov Chain



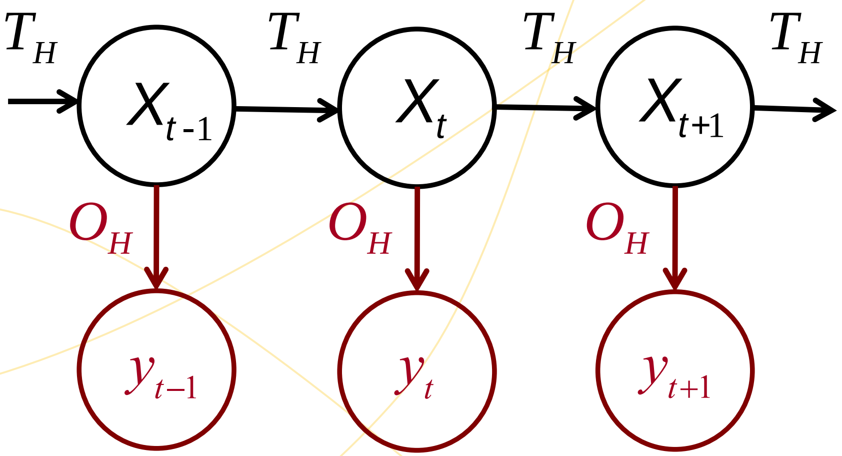
Learning curve with Matlab HMM Toolbox



State Estimation (IS)

Given

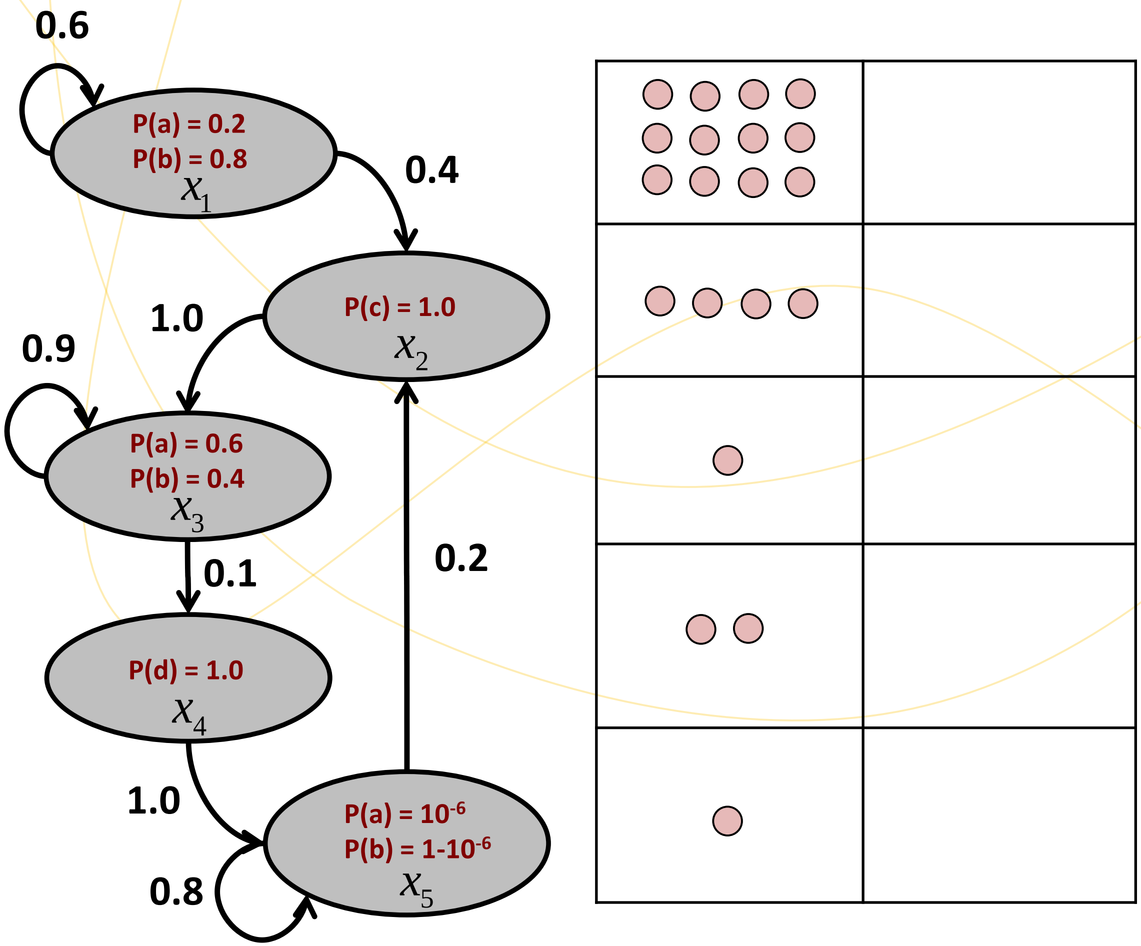
- $\mathbf{P}(X_{t+1} \mid X_t) = T_H, \mathbf{P}(X_1)$
- $\mathbf{P}(y_t \mid X_t) = O_H$
- trace $y_{1,t+1} = y_1, \ldots, y_{t+1}$



Compute $\mathbf{P}(X_{t+1} \mid y_{1:t+1})$

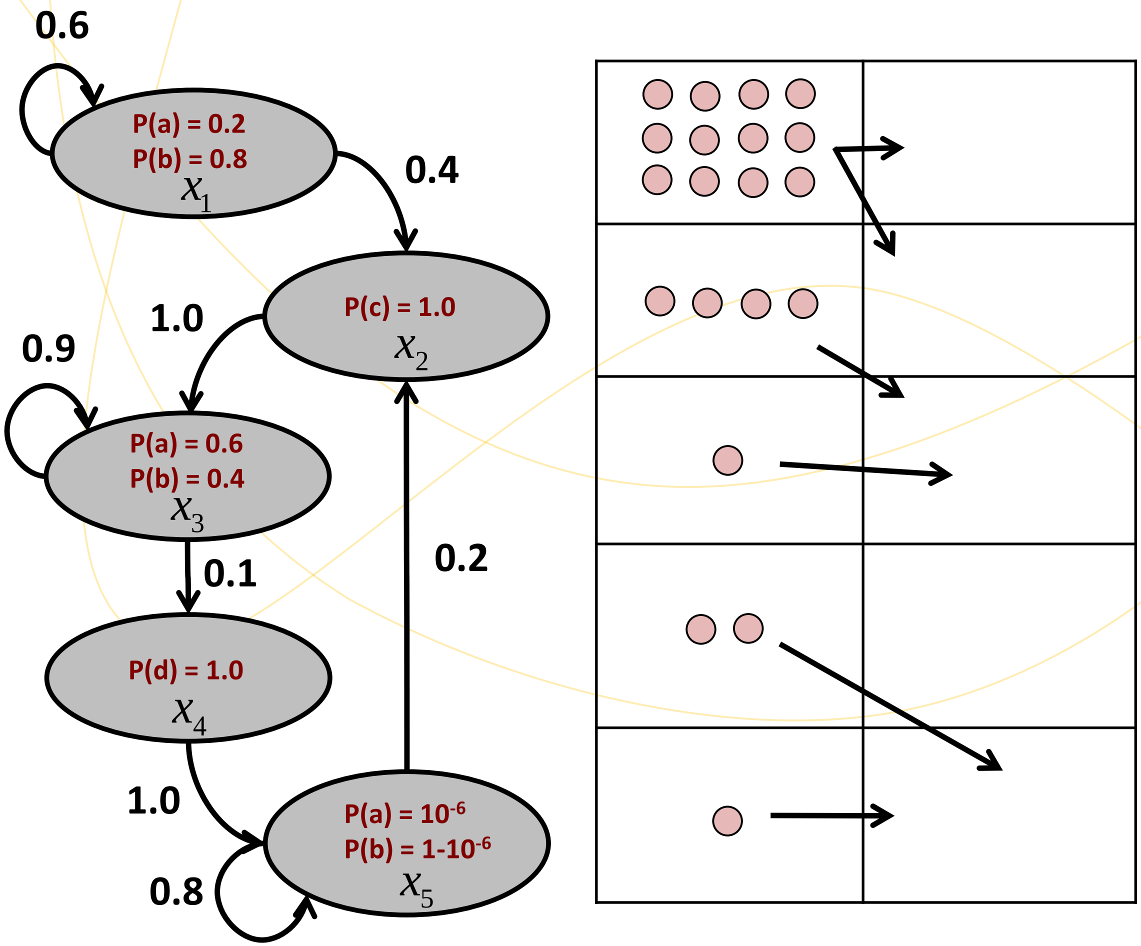
State Estimation (IS)

Initial distribution of the particles



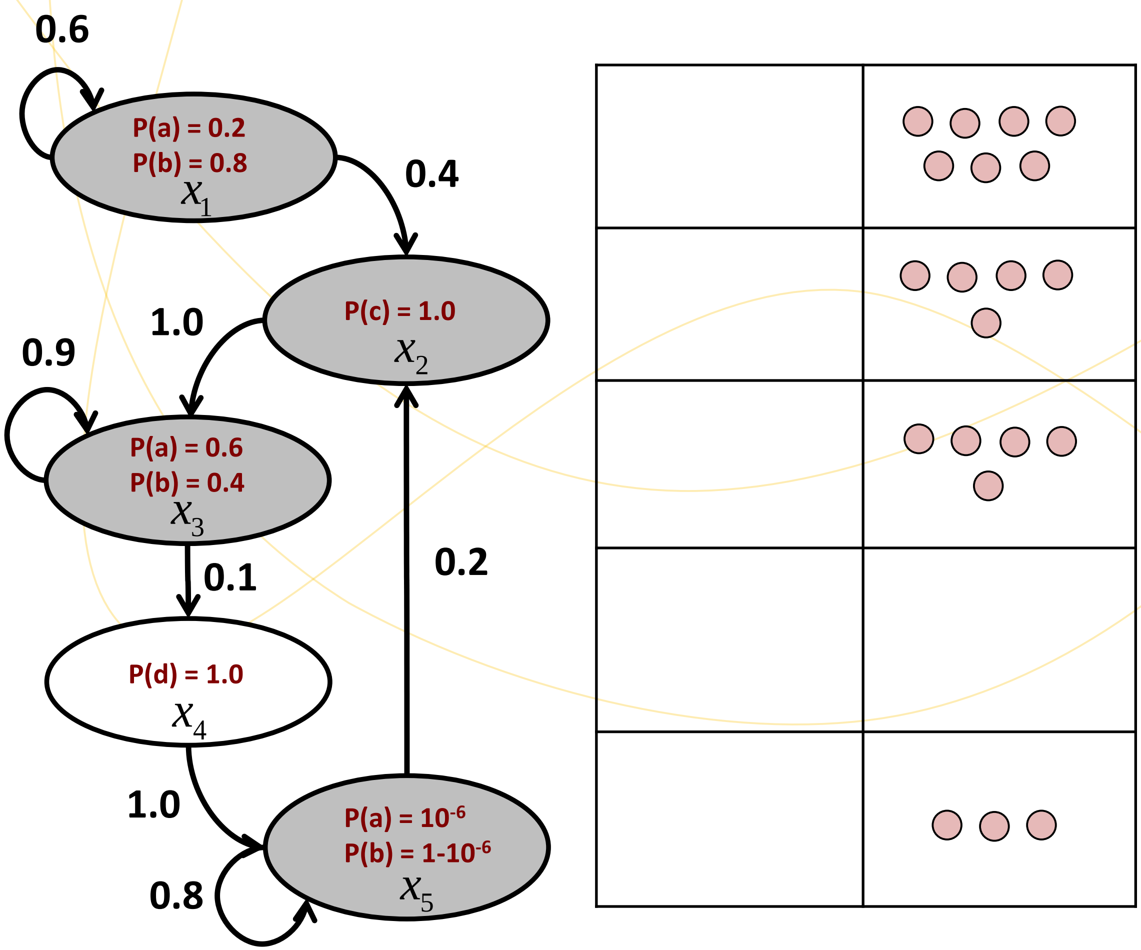
State Estimation (IS)

Simulate the CPS

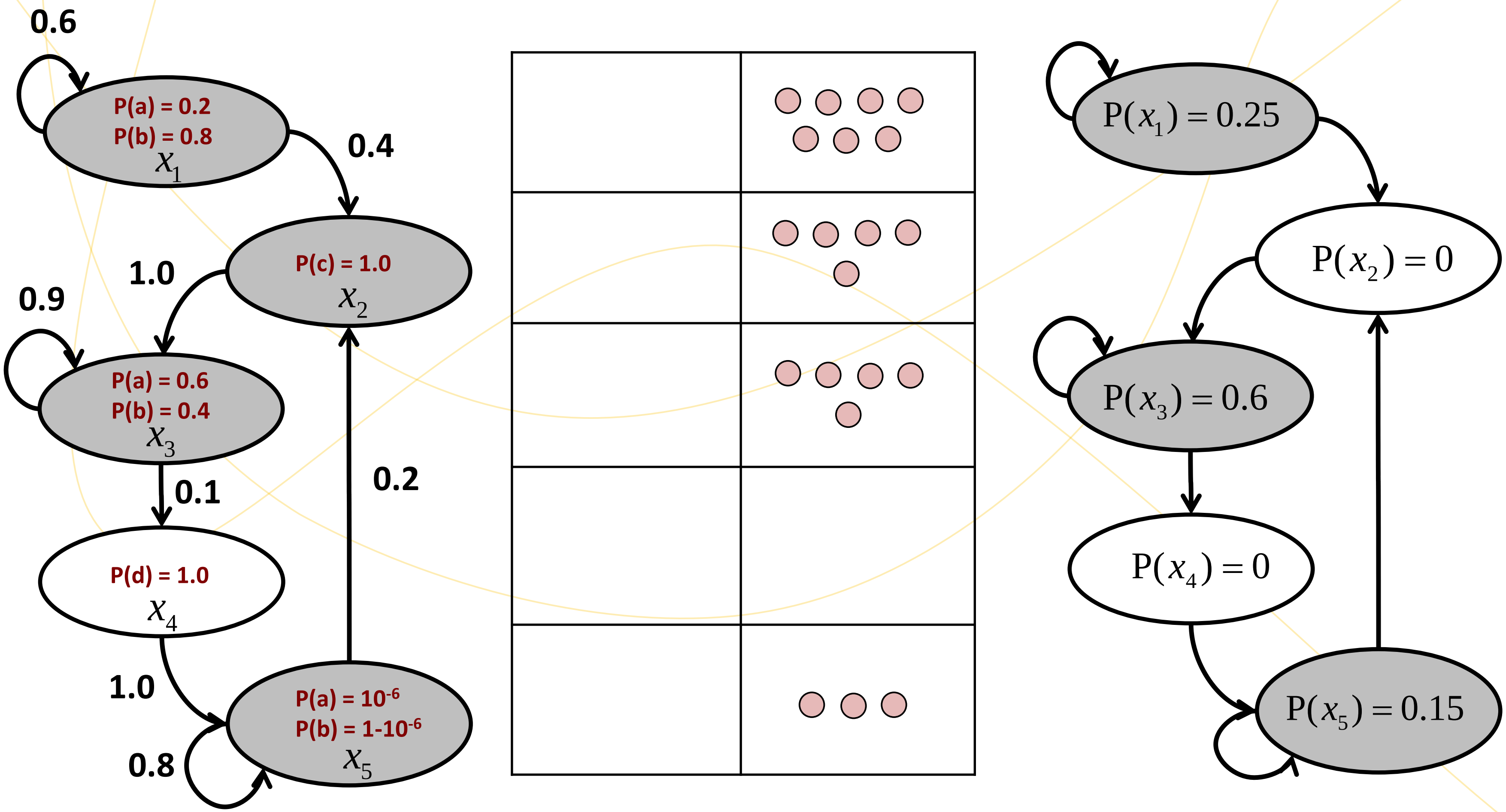


State Estimation (IS)

New configuration of the particles



Observe ‘a’ and resample the particles



Property Decomposition

A nested sequence of temporal logic properties:

$$\varphi_0 \Leftarrow \varphi_1 \Leftarrow \varphi_2 \ldots \Leftarrow \varphi_n = \varphi$$

A set of increasing levels: $0 = \ell_0 < \ell_1 < \ell_2 < \ldots < \ell_n = T$

- **Reaching a level implies having reached all the lower levels:**

$$\mathbf{P}(\ell \geq \ell_i) = \mathbf{P}(\ell \geq \ell_i \mid \ell \geq \ell_{i-1})\mathbf{P}(\ell \geq \ell_{i-1}),\ \mathbf{P}(\ell \geq \ell_0) = 1,\ \gamma = \mathbf{P}(\ell \geq \ell_n)$$

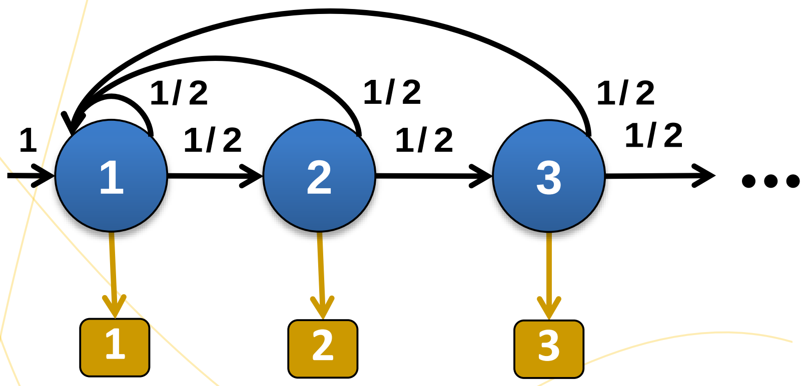
- **The shorter trace satisfying more intermediate properties is given a higher score**

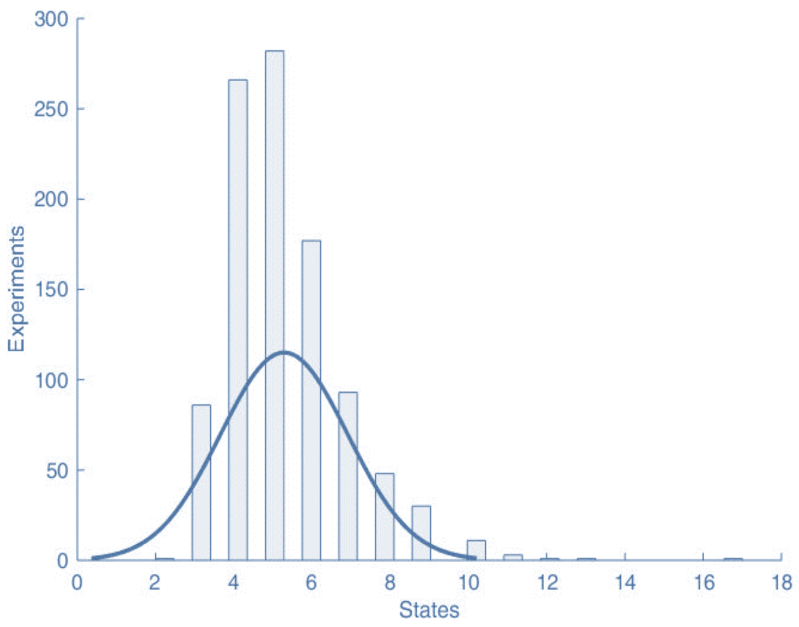
The probability of the rare event:

$$\gamma = \prod_{i=0}^{n} \mathbf{P}(\ell \geq \ell_i \mid \ell \geq \ell_{i-1})$$

- **Levels are chosen such that to minimize the relative variance of the final estimate**

Adaptive Levels for Control (ISp)



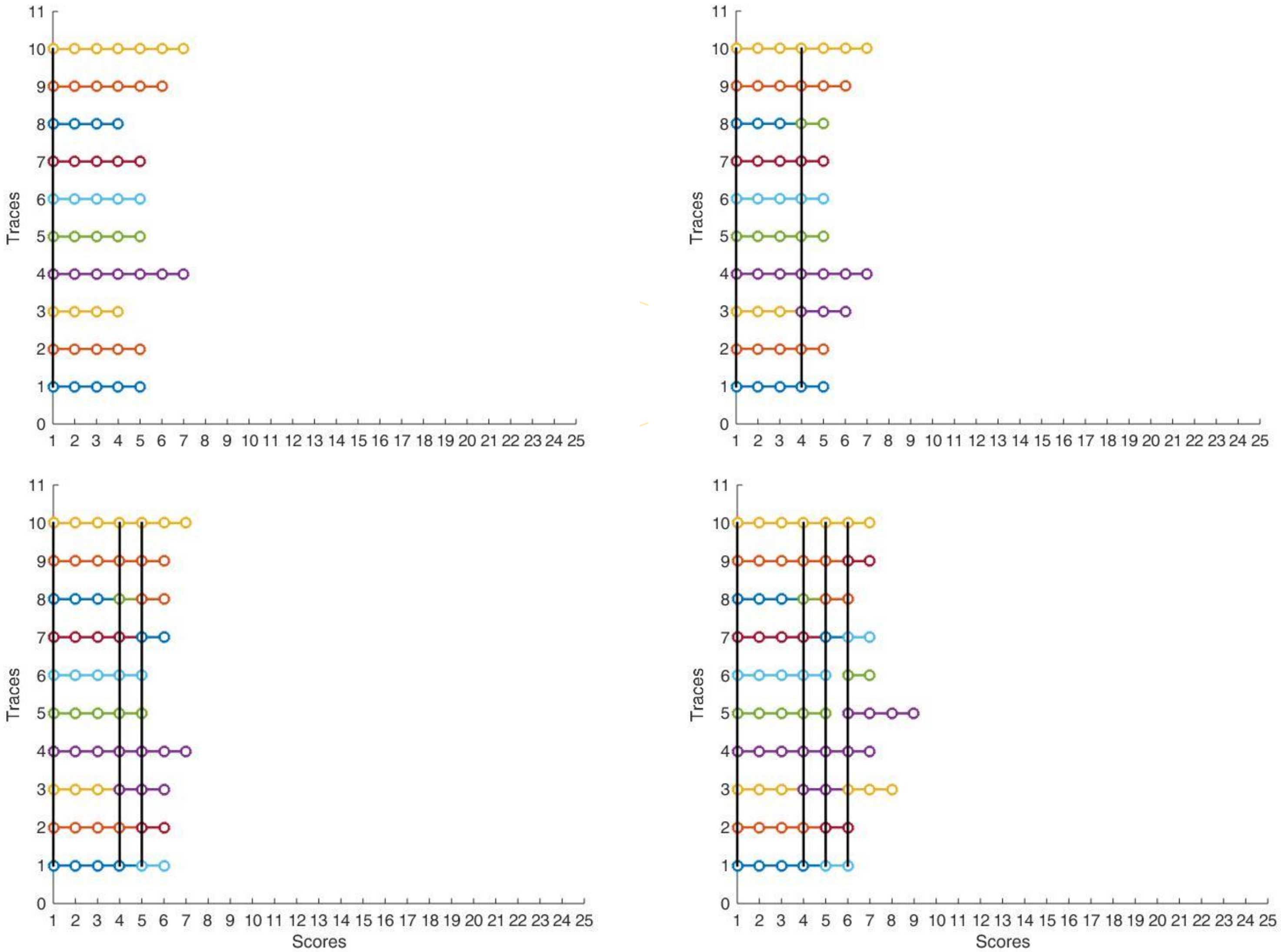


Check the property of reaching state N within N-1 transitions

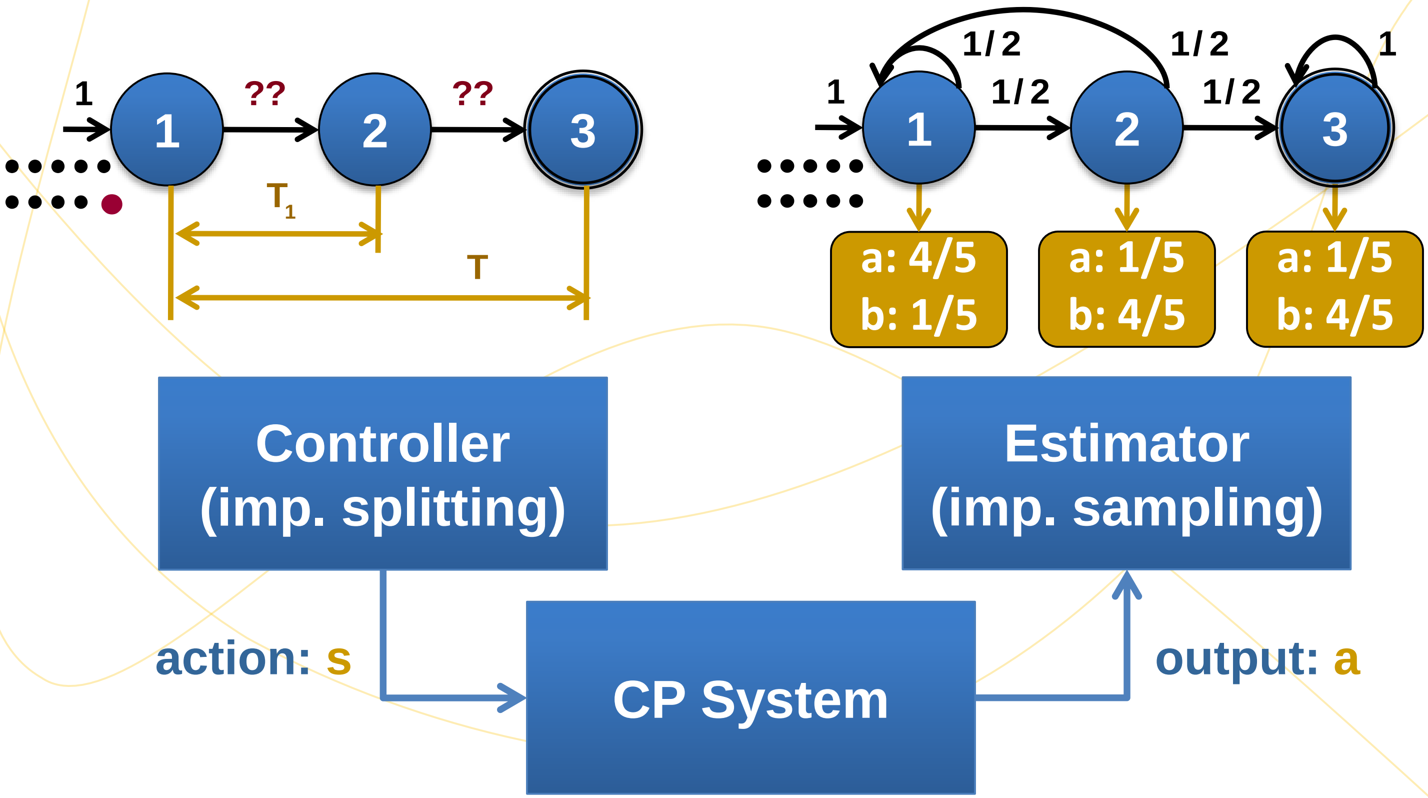
$$T_H = \begin{bmatrix} 1-p & p & 0 & \dots & 0 \\ 1-p & 0 & p & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 1-p & \dots & \dots & p & 0 \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix} \qquad O_H = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 1 & 0 \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Adaptive Levels for Control (ISp)

Simulation with 10 particles for checking the property of reaching state 25



Model Checking as Feedback Control



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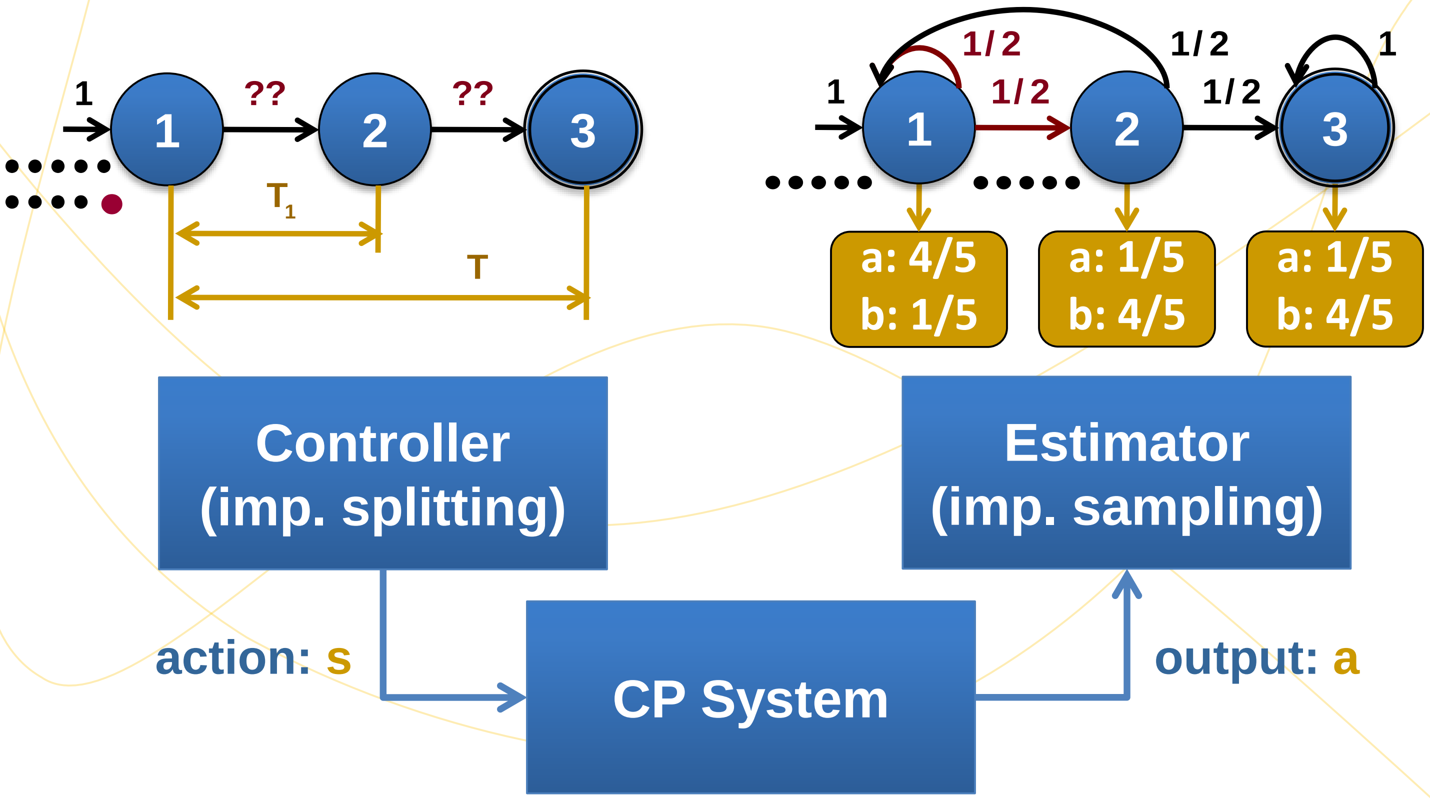

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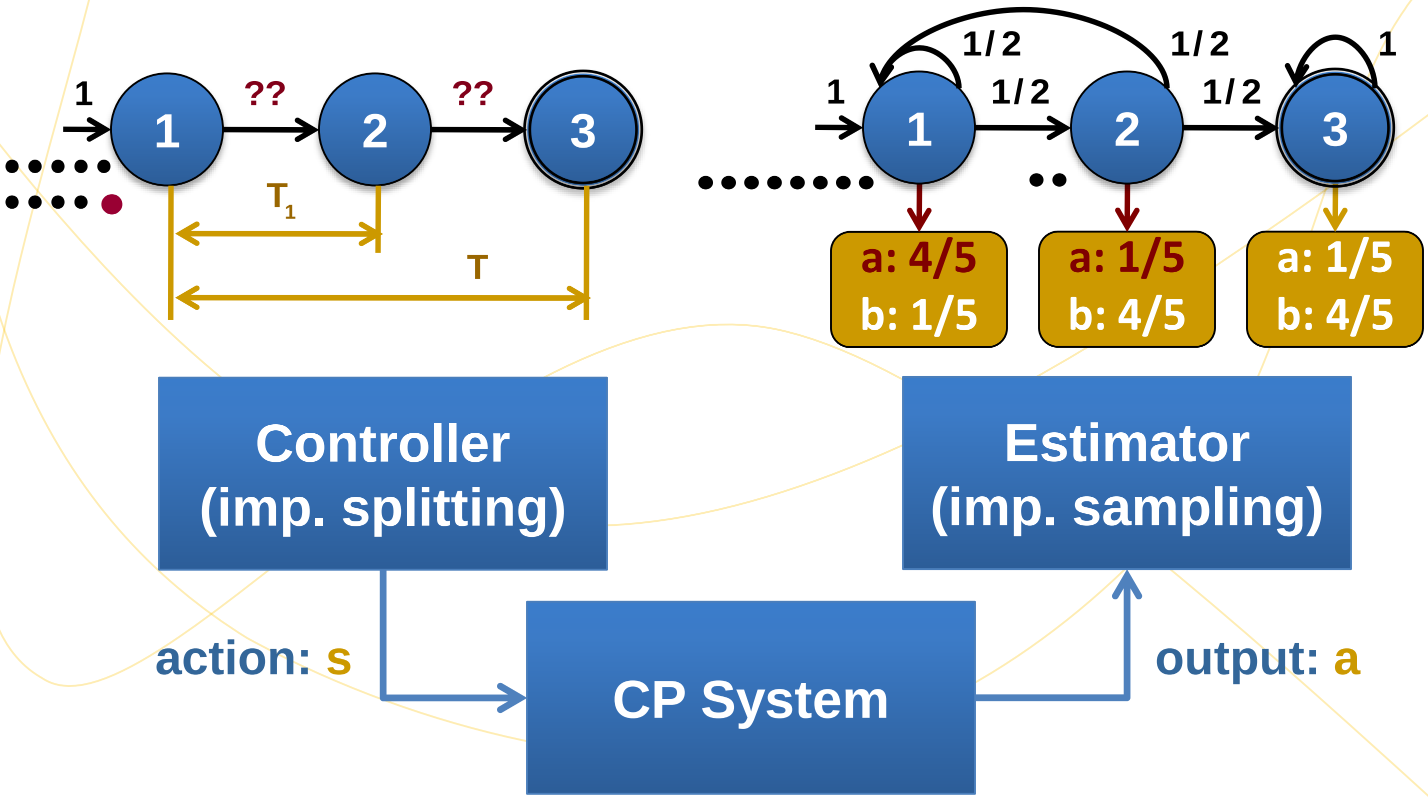

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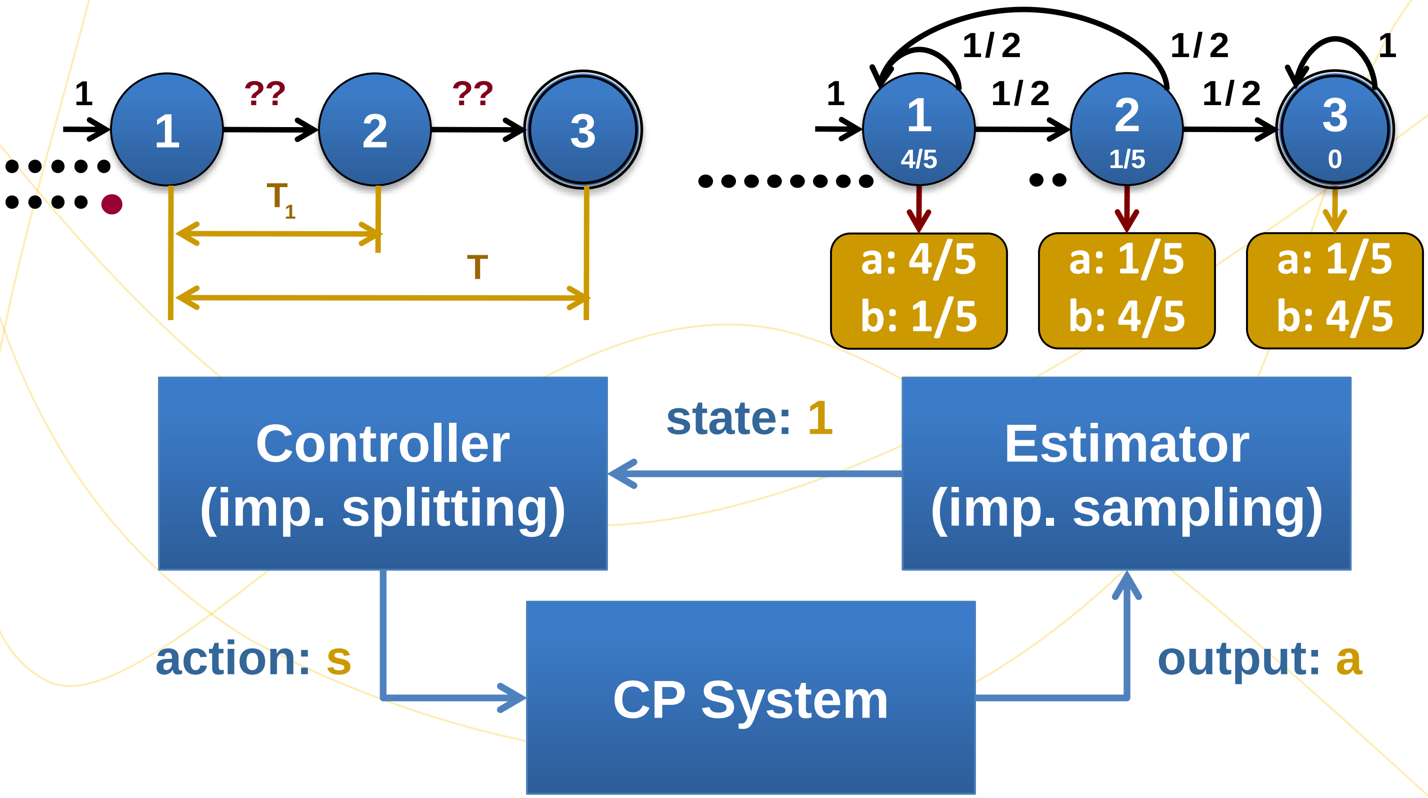

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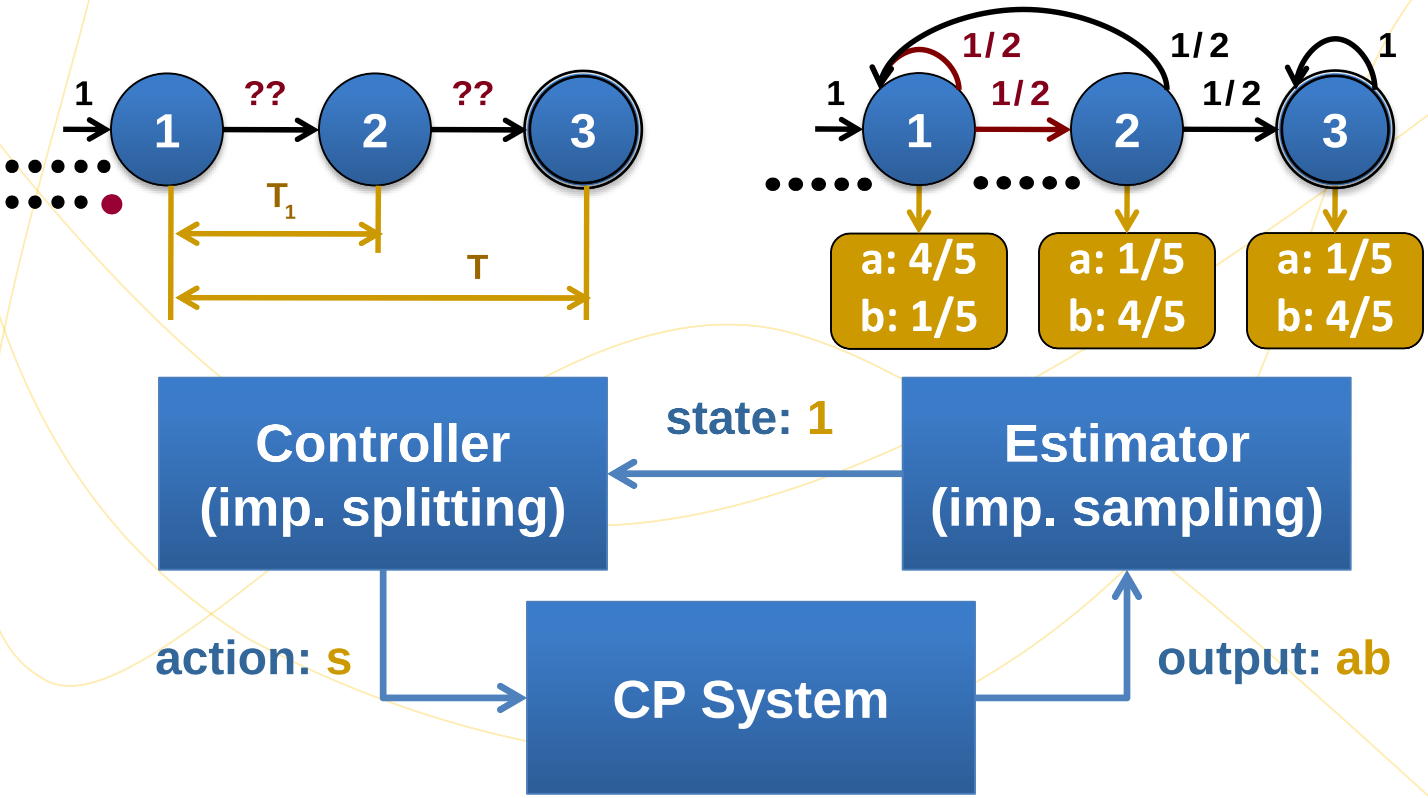

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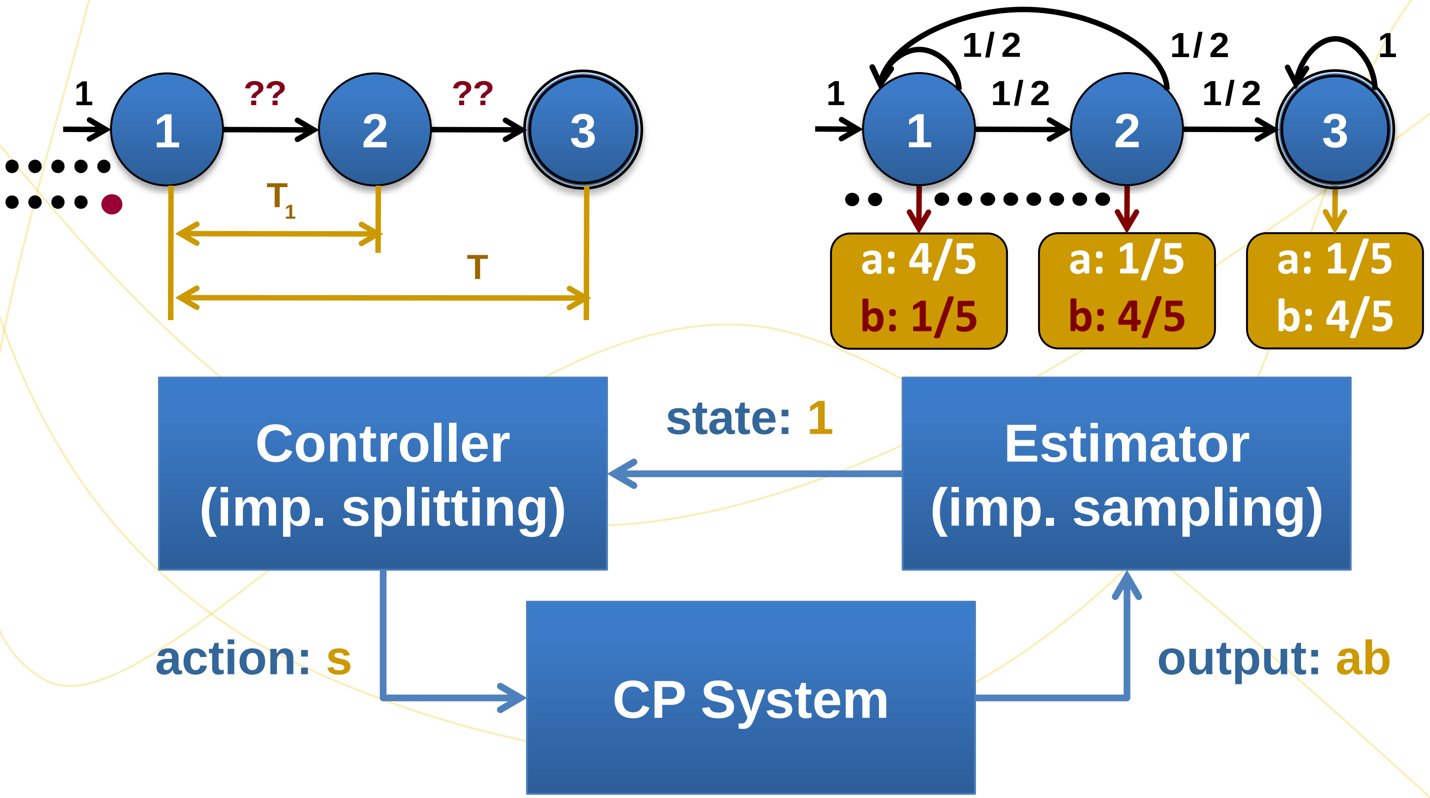

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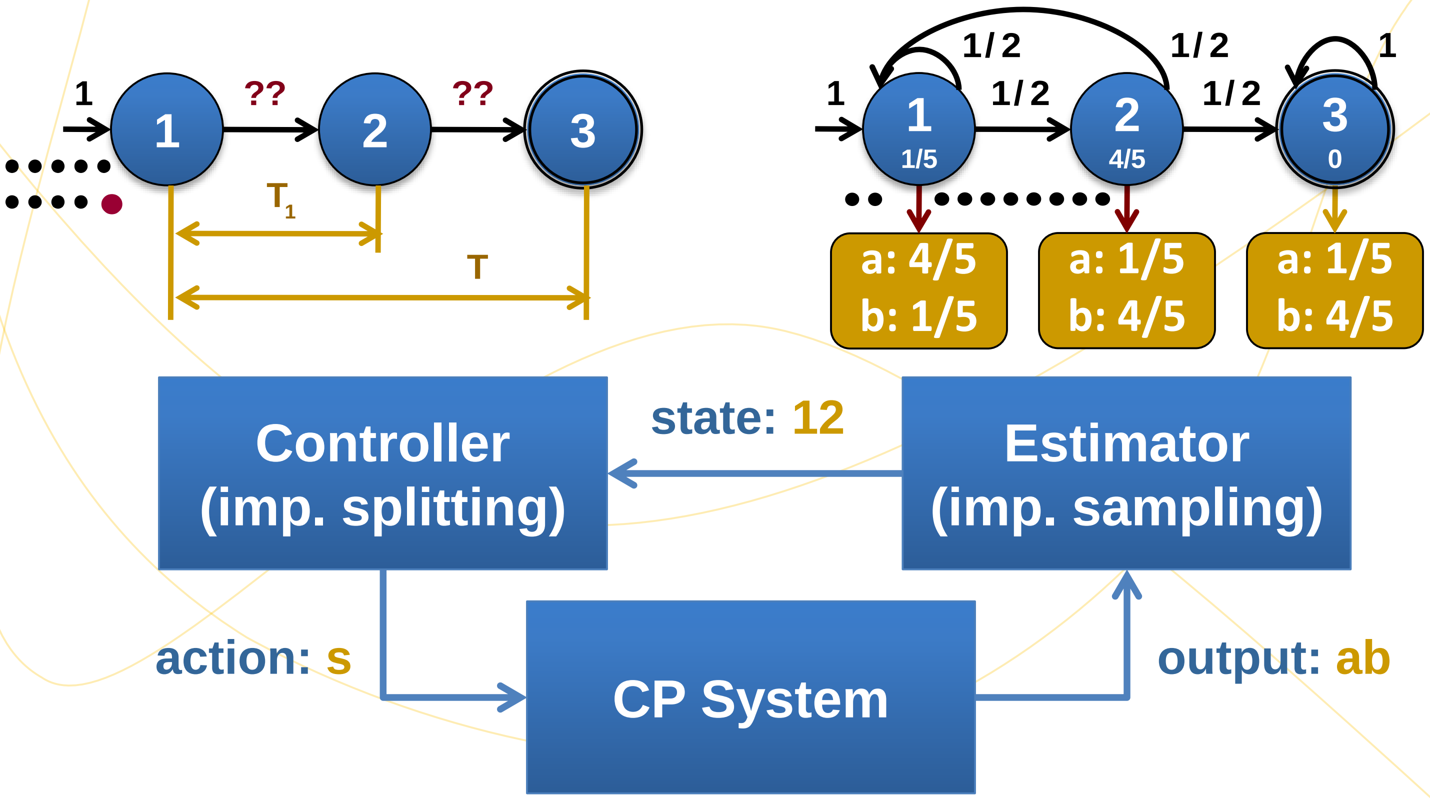

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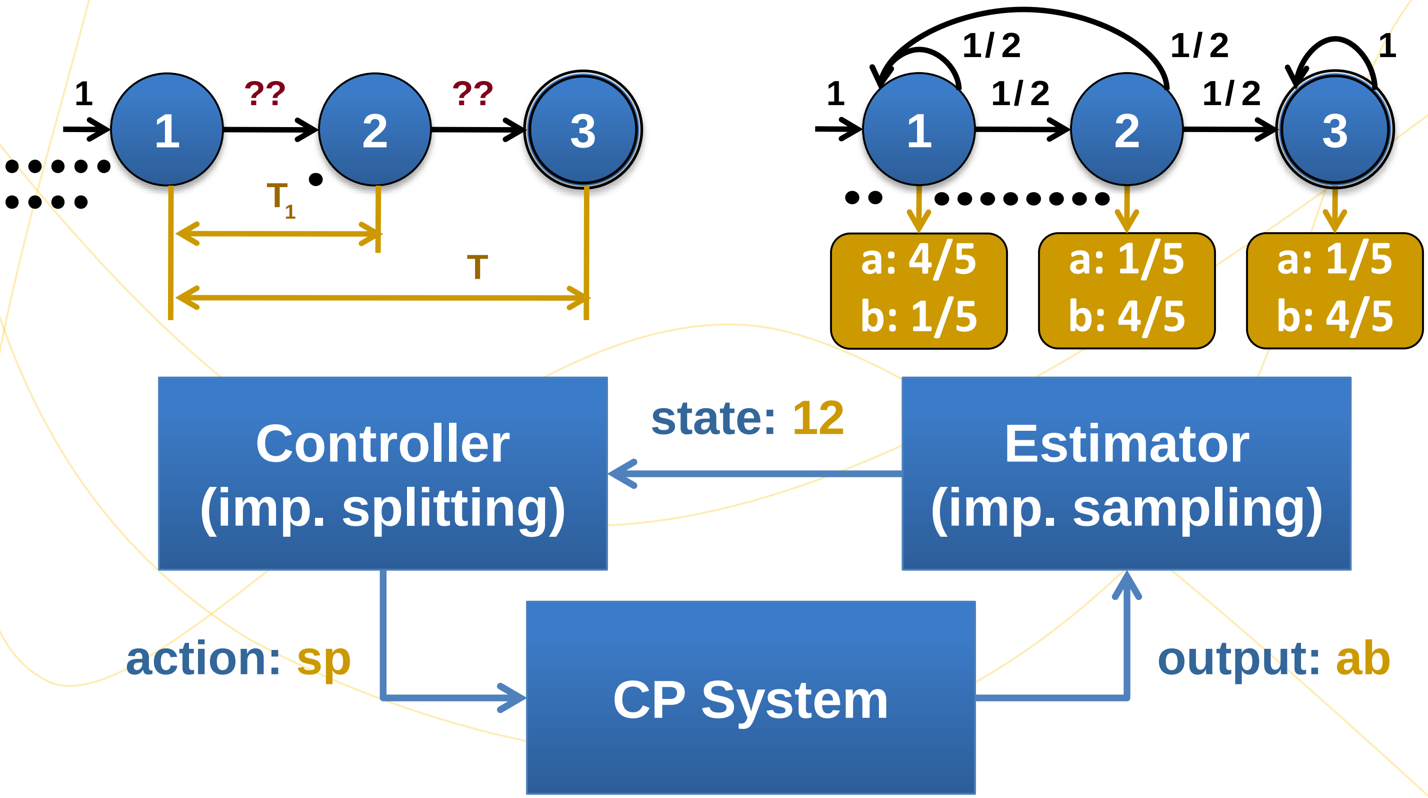

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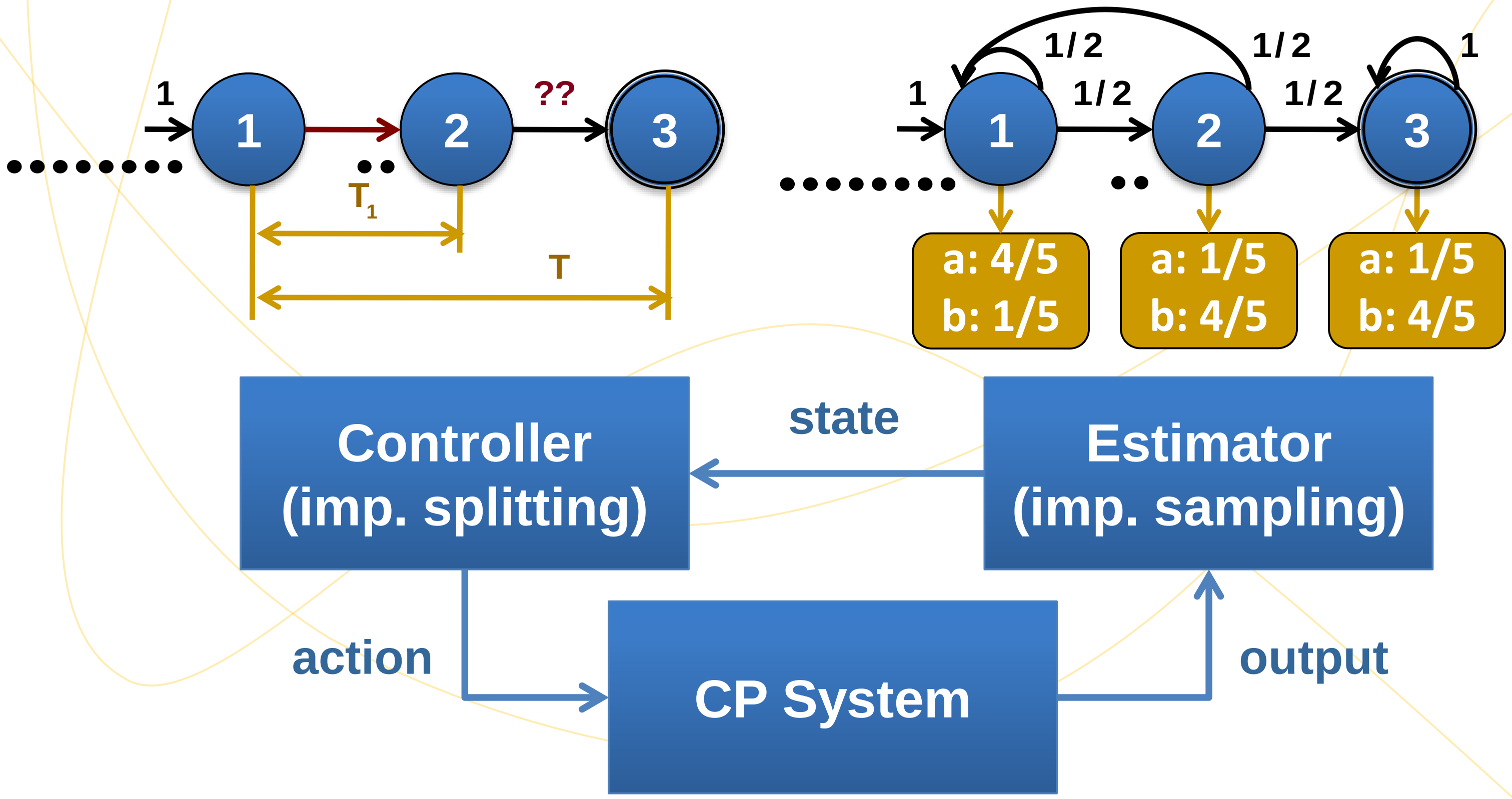

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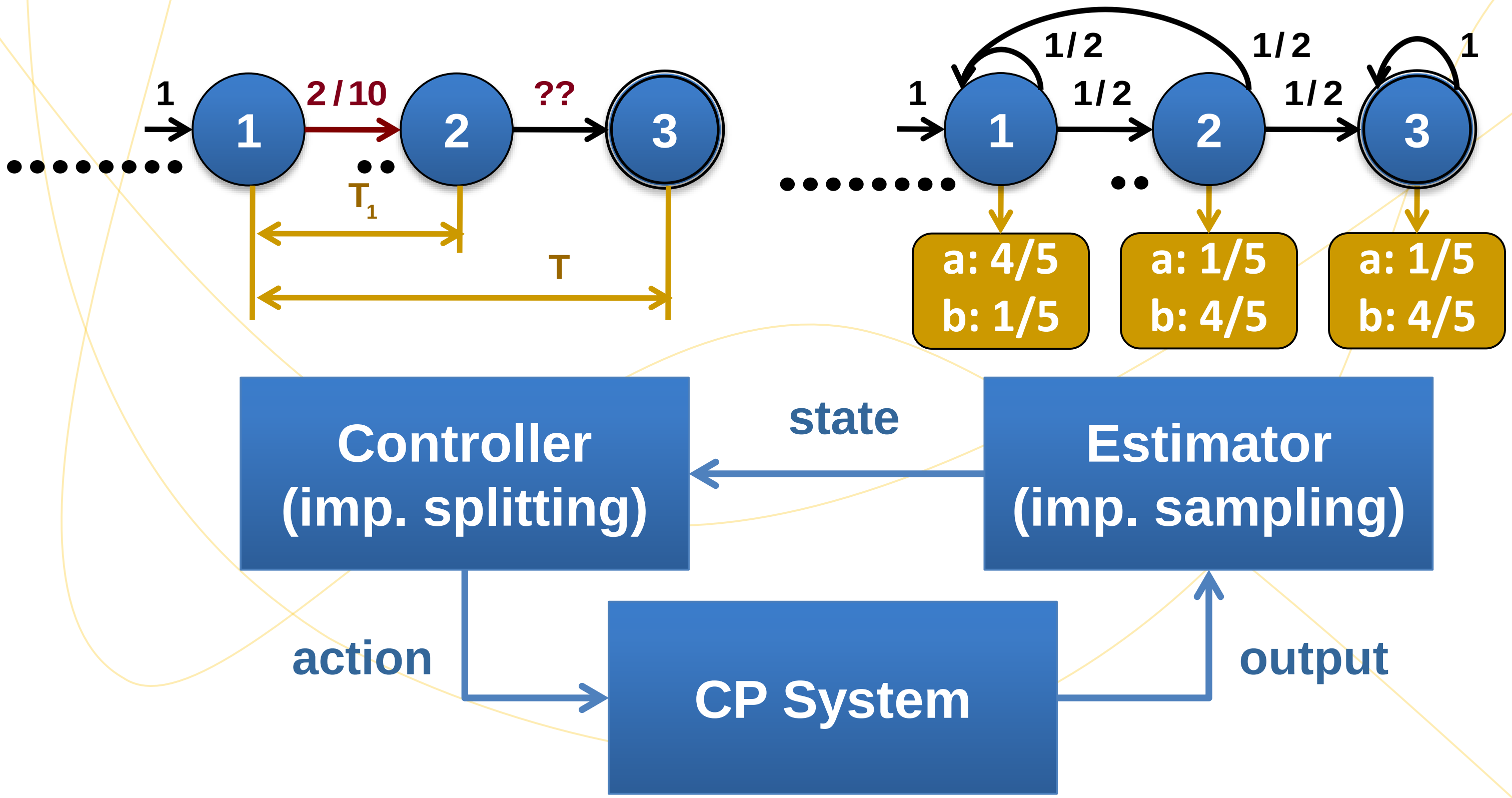

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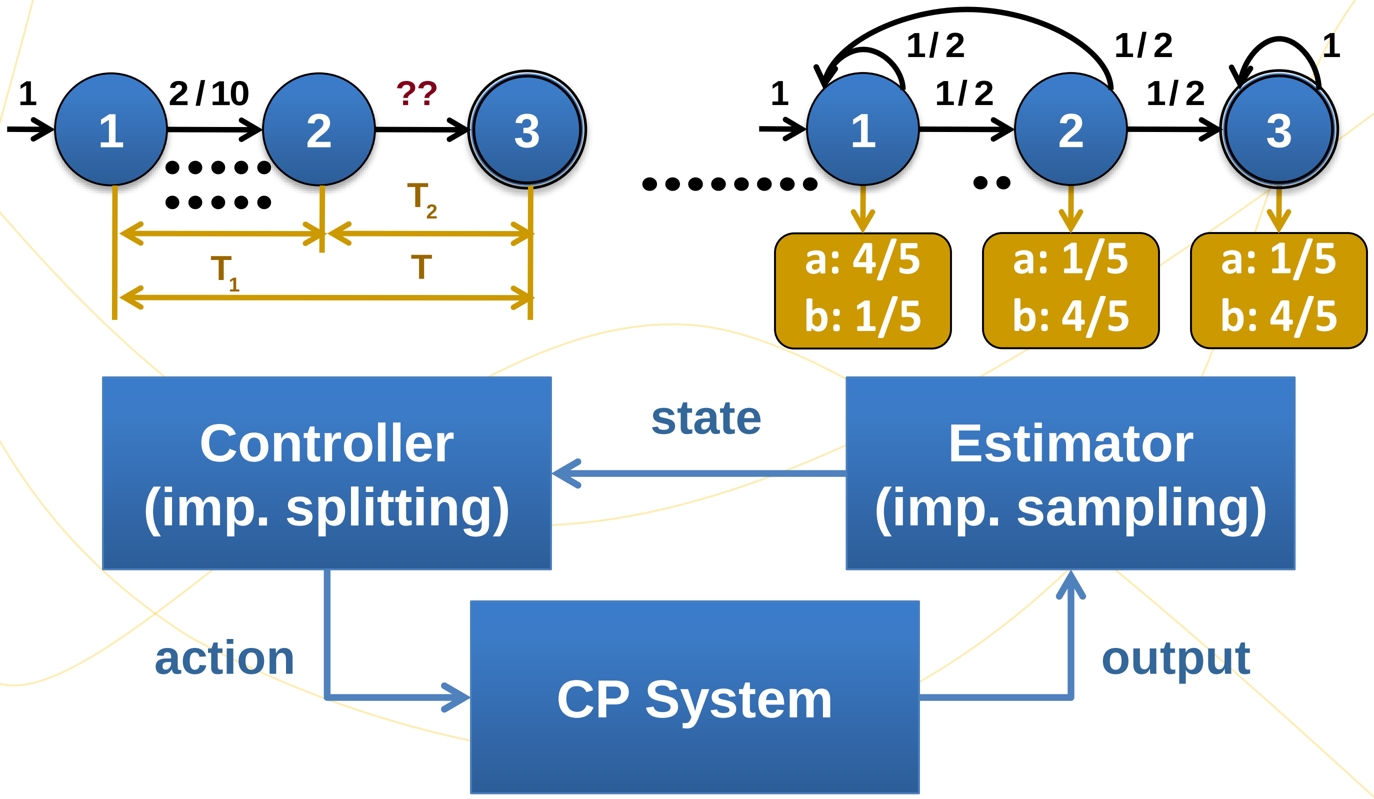

Future Research

- Testing on real case studies of CPS
- Efficient scoring for importance splitting
- Optimal derivation of the levels for importance splitting
- Importance sampling gives the beliefs and not actual states
- Optimal control from the belief-states

Thank you!